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NEW APPROACH FOR MULTI DISCIPLINARY DESIGN OPTIMIZATION WITH MODEL MANAGEMENT FRAMEWORK

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Abstract. *For multi-disciplinary optimization, we present a three level variables elimination approach involving, system level design variables Z, output variables Y_i of discipline i, for interaction with other disciplines and private variables X_i of discipline i. We first eliminate the private variables X_i by using meta models for each discipline. The output variables Y_i of discipline i are optimal with respect to its private variables X_i, for discipline specific objective function. Then an equilibrium system between interaction variables Y_i is solved. This system is viewed as state equations. This allows elimination of interaction variables Y_i using classical apporoches like adjoint method. The main advantage of this approach is to accurately satisfy the state equation of the system. After elimination of interaction varibles, a system level optimization problem restricted to variables Z, is solved. The reliability of each meta model is cheked at optimal point. If the optimality point is on the boundary of the validity domain of particular meta model, then a new meta model for that particular discipline centered around the optimal point is created and the optimization cycle is reiterated with new meta model. We demonstrate our apporach for two MDO test cases, problem of combustion of Propane and design of supersonic business jet.*

1 INTRODUCTION

The goal of MDO is to define a methodology for disciplines interaction with the aim of solving a global design problem. Optimization is only one part of the MDO process.The other inseperable part of MDO process is the framework within which different disciplines interact and share information. A good MDO framework must allow sufficient disciplinary autonomy and should align indivisual disciplinary objectives to the gloabl objectives. Different frameworks for interaction amongst subsystem and system level tasks gives rise to different MDO methods like All At Once (AAO), Multi Disciplinary Feasible (MDF), Indivisual Discipline Feasible (IDF), Bi-Level Integrated System Synthesis (B-LISS), Collaborative Optimzation (CO), etc.. Number of survey articles provide details of different MDO methods and their comparison.[1,2,3]

In this paper, we provide an alternative approach for MDO. It differes from existing MDO strategies by its methodology to handle different design parameters involved in the MDO process. Our approach for MDO is described in section 2. Comparision between "METHOD NAME" and existing methods is discussed in section 3. Finally, application of " Method NAME " is presented for two MDO test cases, namely, the problem of combustion of Propane[5] and the design of supersonic business jet.[6]

2 MDO USING " Method NAME " APPROACH

The common point between different publications is to consider two types of design parameters, public parameters (shared by disciplines) and private parameters (specific to the given discipline). Unlike most contributions, we consider only public parameters. The " Method NAME " framework assumes that for a given choice of public parameters, private parameters are fixed by each discipline at their optimal value. Private parameters are not visible outside the specific discipline. This allows complete autonomy to indivisual disciplines to handle its disciplinary variables. We denote private variables of i^{th} discipline by X_i. The public paramters are further divided into two: interaction variables, Y_i and system level variables, Z. Interaction variables Y_i are outputs of i^{th} discipline and are shared by other disciplines for disciplinary analysis and optimzation. So in " Method NAME " approach design variables are divided into three groups. These groups are handled at two levels namely, the disciplinary or subsystem level and the system level.

2.1 The disciplinary level

The disciplinary level is responsible for the construction of meta models. Each discipline is supposed to construct a meta model that depends only on the public parameters. Also, for each set of public parameters $p_u = \{Z, Y_i\}$, the response of the meta-model is optimal with respect to it's private parameters X_i. The meta model can be constructed using techniques like neural network, kringing, reduced order models, etc. If required one can replace the meta model by the real model. The response also includes description about the reliability of the model. Each discipline also provides some constraints on the public parameters to indicate the domain of validity of the meta model. To simplify the presentation, we limit ourselves to two disciplines, however the generalization to n disciplines is straight forward.

Consider two disciplines represented by their meta models A_1 and A_2.

$$a_1 = A_1(p_u, a_{21}) \tag{1}$$

$$a_2 = A_2(p_u, a_{12}) \quad (2)$$

The vector a_i represents the output of the i^{th} discipline. Vectors a_{12} and a_{21} represent the coupling between the two disciplines. It is convinient to write the output of the meta model A_1 into two parts, namely,

$$a_{11} = A_{11}(p_u, a_{21}) \quad (3)$$
$$a_{12} = A_{12}(p_u, a_{21}) \quad (4)$$

or in general, the output of the i^{th} discipline can be written as

$$a_i = (a_{ii}, a_{ii'}) = (a_i^{fc}, a_i^c, a_i^f, a_{ii'})$$

where,

a_{ii}	is the output of discipline i to the system level optimizer.
$a_{ii'}$	is the vector of coupling variables between discipline i and i'.
a_i^{fc}	is the scalar value of the disciplinary objective function.
a_i^c	is a vector of the disciplinary constraints of the type ($a_i^c \leq 0$).
a_i^f	is the value of the reliability criteria for meta model approximation.

Note for the present case $i' = 3 - i$.
Each discipline transmits following information to the system level.
- The meta model A_i in the form of a portable program.
- Constants b_i^i, b_i^s which indicate bounds on the system level design variables.

These bounds ensure the validity of meta models for given system level variables. Superscripts i and s indicate lower and upper bound respectively. Bound constraints are added to the model reliability constraints a_i^f obtained from the output of the meta model and are of the form $a_i^f \leq 0$.

2.2 System level

At system level we first eliminate interaction variables $Y_i = \{a_{ii'}\}$ and then solve optimization problem with respect to variables Z. The elimination approach for interaction variables depends on the type of coupling between different subsystems.

2.2.1 Strong coupling

The coupling implies a coupling between the disciplinary solvers. When coupling between disciplinary solvers is strong, it is more reasonable to regroup these two disciplines into one.

2.2.2 Weak coupling

Consider the terms in equations (1) and (2) representing coupling between the disciplines.

$$a_{12} = A_{12}(p_u, a_{21}) \tag{5}$$
$$a_{21} = A_{21}(p_u, a_{12}) \tag{6}$$

The system 5, 6 can constitute a triangular system and then is solved either by forward or backward substituion. If the system 5, 6 is not triangular, it is solved in an iterative manner. In principle, this set of equations admits an unique solution. The lack of unique solution indicates the failure of the meta model. For a given set of public parametres, one solves

$$\min_{a_{12},a_{21}} ||a_{12} - A_{12}(p_u, a_{21})||^2 + ||a_{21} - A_{21}(p_u, a_{12})||^2$$

The crucial point, compared to classical methods is to give priority to the solution of the system described by 5, 6. Eventually, equations 5, 6 can be considered as state equations. This permits us to exploit classic tools such as adjoint methods available to optimal desgin. The use of adjoint method to solve state equation is more rewarding when number of interaction variables are large. These techniques then eliminate all the interaction variables Y_i. With the elimination of interaction variables, the system level optimization problem is then limited only to public variables Z. This makes the optimzation problem small and notably global in nature. This is a prime aspect of " Method NAME " at which it differes from the existing methods.

2.3 System level optimization problem

At system level, the optimization problem is

$$\begin{cases} \min_Z f(Z) = (\lambda_1 * a_1^{fc} + \lambda_2 * a_2^{fc}) \\ a_i^c \leq 0, i = 1, 2, \text{ disciplinary constraints} \\ a_i^f \leq 0, \text{ indicator for meta model reliability} \\ b_i^i \leq Z \leq b_i^s, \text{ bounds on } Z \text{ for meta model validity} \\ + \text{ system level constraints.} \end{cases} \tag{7}$$

where, λ_i are weights of the subsystem objective function.

When the optimal solution to above problem is obtained, " Method NAME " checks the model reliability for solution point and location of the solution point with respect to the boundary of the validity domain of each meta model. If the obtained solution is on the boundary of the validity domain of any of the meta models, it asks for an update in the meta model such that the new construction of a disciplinary meta model is centered around this point. With this updated meta-model it restarts the process of optimization. Meta model management framewrok is thus embedded in our approach. It permits the optimizer to move from one design space to another based on the indivisual desciplines reliability constraints.

The " Method NAME " approach for MDO is schematically shown in Figure 1.

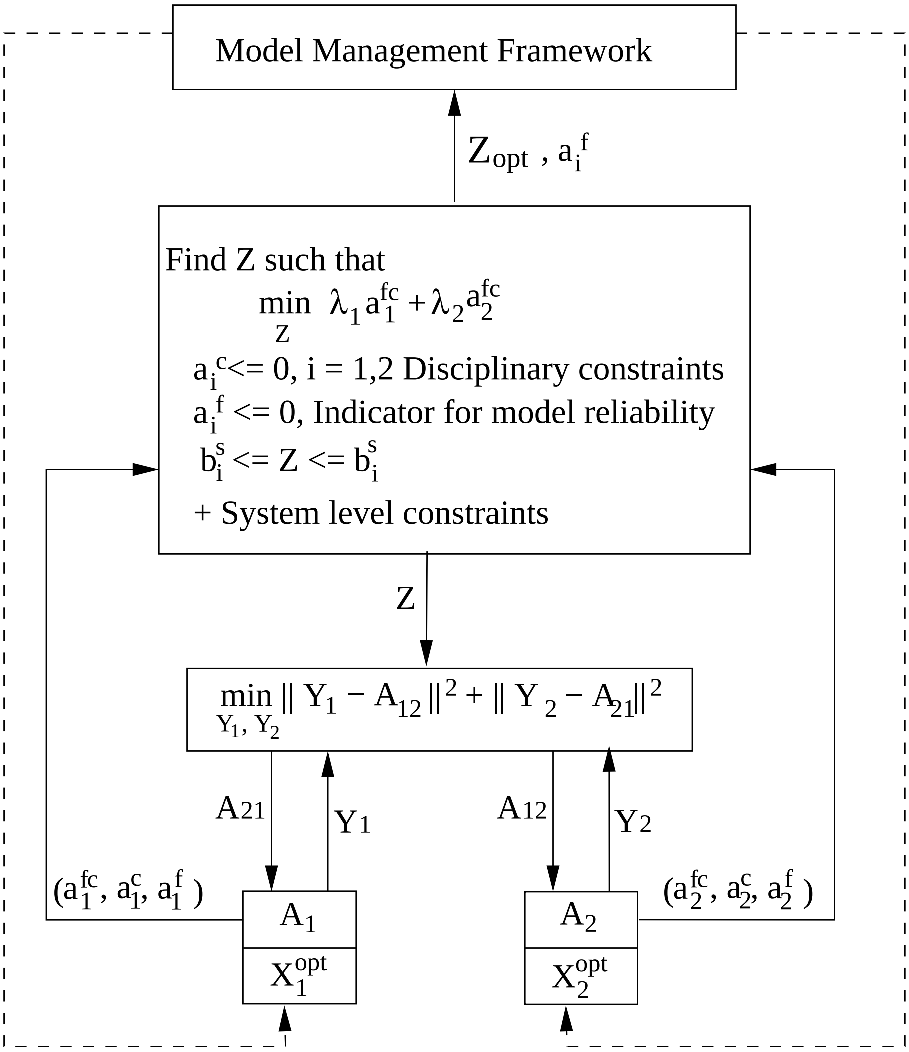

Figure 1: "Method Name" approach for MDO

3 COMPARISON WITH OTHER METHODS

The approach presented in section 2 practically encompasses all known MDO methods:

- The proposed method is inspired extensively by the BLISS method (Bi - Level Integrated System Synthesis) [4]. It differes from BLISS in the way it handles public and private variables. " Method NAME " permits complete autonomy to indivisual disciplines to handle its private parameters and to decide indivisual disciplinary objectives. Unlike the original version of BLISS[4] this method does not require evaluation of system sensetivity equations and thus the choice of sub-sytem objective function is left to individual disciplines.
- MDA (Multi - Disciplinary Analysis), the proposed method becomes the MDA

method, when meta models are replaced by the real models.

- AAO (All At Once), the method proposed in this paper becomes the AAO method if one replaces meta models by the real models and that one solves equations (5), (6) and (7) simultaneously.

- CO (Collaborative Optimization), the CO method puts the equations (5) and (6) as a constraint on the system level optimization.

However, all these methods in a way rely on fixed point iteration strategy to determine interaction variables and is not roboust. Our approach to view system of equations governing interaction variables as state equations is more roboust and even efficient if number of interaction variables are large. Moreover, none of the above method propose any strategy for describing the quality of optimal solution. The meta model management framework we proposed permits to evaluate the accepatability criteria for optimal solution based on model reliability and model validity constraints.

4 APPLICATIONS

We illustrate application of " Method NAME " algorithm to two MDO test cases, namely, problem of combustion of Propane[5] and conceptual design of super sonic busineess jet[4]. These simulations were carried out using MATLAB optimization toolbox. MATLAB function *FMINCON* which is SQP based optimization solver is used in these studies [6]. The primary aim of these studies is to show the accuracy with which the state equations of the system are satisfied by our approach. The meta models used in these studies does not permit to demonstrate strategy we proposed for meta model management. This aspect of our approach will be dealt elsewhere.

4.1 Problem of combustion of Propane

4.1.1 Description

This test case is obtained from NASA MDO test suite website[5] and the problem description is reproduced here for completeness. This chemical equilibrium problem describes the combustion of Propane in air. Products of combustion are denoted by $x_i, i = (1 \text{ to } 10)$ and are unknowns to be determined. They represent the number of moles of each product formed for each mole of Propane burned. The 11^{th} unknown x_{11}, used to simplify equations, is essentially the sum of the other 10 unknowns. There are 11 equations denoted by $f_i, i = (1 \text{ to } 11)$ relating products of combustion. The equation denoted by f_{11} is the requirement that the sum of the first 10 unknown (x) equals the 11^{th} unknown x_{11}. Fixed parameters are P (pressure in atmospheres) R (the air to fuel ratio) and $K_i, (i = (5 \text{ to } 10)$ denote measured data. Ideally we want all the $f(i)$'s $(i = 1 \text{ to } 11)$ to be zero and all the x's must be greater than zero. The set of equations governing the combustion process are given by equation 8

$$
\begin{aligned}
f_1(x) &= x_1 + x_4 - 3 \\
f_2(x) &= 2x_1 + x_2 + x_4 + x_7 + x_8 + x_9 + 2x_{10} - R \\
f_3(x) &= 2x_2 + 2x_5 + x_6 + x_7 - 8 \\
f_4(x) &= 2x_3 + x_9 - 4R \\
f_5(x) &= K_5 x_2 x_4 - x_1 x_5 \\
f_6(x) &= K_6 x_2^{1/2} x_4^{1/2} - x_1^{1/2} x_6 \left(\frac{P}{x_{11}}\right)^{1/2} \\
f_7(x) &= K_7 x_1^{1/2} x_2^{1/2} - x_4^{1/2} x_7 \left(\frac{P}{x_{11}}\right)^{1/2} \\
f_8(x) &= K_8 x_1 - x_4 x_8 \left(\frac{P}{x_{11}}\right) \\
f_9(x) &= K_9 x_1 x_3^{1/2} - x_4 x_9 \left(\frac{P}{x_{11}}\right)^{1/2} \\
f_{10}(x) &= K_{10} x_1^2 - x_4^2 x_{10} \left(\frac{P}{x_{11}}\right) \\
f_{11}(x) &= x_{11} - \sum_{j=1}^{10} x_j
\end{aligned} \tag{8}
$$

4.1.2 MDO formulation

The problem of combustion of Propane is formulated as MDO problem with system level optimization problem as defined in Eq. 9. Unknowns x_1, x_3, x_6, x_7 are treated as system level variables whereas $x_2, x_4, x_8, x_9, x_{10}$ are treated as interaction variables. There are three subsystems and no private variables. Table 1 shows the subsystem level problem.

$$
\begin{aligned}
&\min_Z f = (f_2 + f_6 + f_7 + f_9) \\
&\text{subjected to} \\
&\qquad f_2, f_6, f_7, f_9 \geq 0 \\
&\qquad Z \geq 0
\end{aligned} \tag{9}
$$

Details	Input	Output	Constraints
Sub system 1	$Z = \{x_1, x_3, x_6, x_7\}$	$f_2, Y_1 = \{x_2, x_4\}$	$f_1 = 0, f_5 = 0$
Sub system 2	$Z = \{x_1, x_3, x_6, x_7\}$	$f_6, Y_2 = \{x_8, x_{10}\}$	$f_8 = 0, f_{10} = 0$
Sub system 3	$Z = \{x_1, x_3, x_6, x_7\}$	$(f_7 + f_9), Y_3 = \{x_5, x_9, x_{11}\}$	$f_3 = 0, f_4 = 0, f_{10} = 0$

Table 1: Details of subsystem input output relationship in the problem of combustion of Propane

4.1.3 Results

Three different sets of initial values are used for system level variables. The case 2 represents starting point close to true solution where as case 3 is with initial value of system variables very far from true solution. The results obtained using our approach are shown in Table 2. Results from reference 5 for same set of initial values are reproduced in Table 3. Comparision of these two tables shows that state equations are sastisfied more accurately in our approach. This also results in lesser number of iterations required at system level. Number of subsytem function evaluations include those required for gradient computations. Values of subsystem function evaluation were not available in reference 5.

Cases	case 1	case 2	case 3
Starting point Z	$\{2.0, 20.0, 0.0, 0.0\}$	$\{1.48620, 18.2890, 0.86022, 0.14533\}$	$\{2.0, 40.0, 5.0, 1.5\}$
Objective function value	3.99677e-008	3.49548e-008	4.06293e-008
No of iterations	5	3	7
No. of system objective function evaluations	30	20	42
No. of calls to subsystem meta-models	2880	1920	4032
Maximum residue from original system	1.50263e-008	1.5112e-008	1.515507e-008
Value of constraint f(2)	6.63687e-009	6.59901e-009	6.8746e-009
Value of constraint f(6)	8.32817e-009	7.39945e-009	8.49799e-009
Value of constraint f(7)	8.45813e-009	7.60243e-009	8.61474e-009
Value of constraint f(9)	1.07169e-008	7.48939e-009	1.08254e-008

Table 2: Results obtained for the problem of combustion of Propane

Cases	case 1	case 2	case 3
Objective function value	5.544805e-4	8.7646e-004	1.6037e-004
No of iterations	50	50	60
No. of system objective function evaluations	551	522	666
Value of constraint f(2)	1.2974e-004	3.5465e-004	5.3219e-005
Value of constraint f(6)	6.9800e-005	7.0957e-005	7.7294e-005
Value of constraint f(7)	3.1334e-004	2.6657e-004	9.7761e-006
Value of constraint f(9)	4.1600e-005	1.8647e-004	2.0084e-005

Table 3: Values reported for the problem of combustion of Propane using code from reference 5

4.2 Design of super sonic business jet (SSBJ)

This test case corresponds to the problem used by NASA to present the BLISS algorithm[4] and provides representative example of aircraft conceptual design. The global objective is to design supersonic businees jet with maximum range. Subsystems include structure, aerodynamics, propulsion and range. The interaction amongst these disciplines is shown in Figure 2. Further details of this test case are referred to reference 4 and are not reproduced here. Table 4 shows list of public, interaction and private parameters as used in our approach. In the present case meta models used in the original BLISS code were used without any modifications. As outlined in section 2, our method assumes that the choice of private parameters is an affair of individual discipline. To facilitate the comparison with BLISS, private parameters of each discipline were set to the optimal values as obtained in BLISS. With this, the results obtained from our approach should match with those produced by BLISS which is indeed a case as seen from Table 5. The difference in approach is that, we have not carried out system sensetivity analysis as in BLISS and have given complete autonomy to individual discipline as to what should be there objective function.

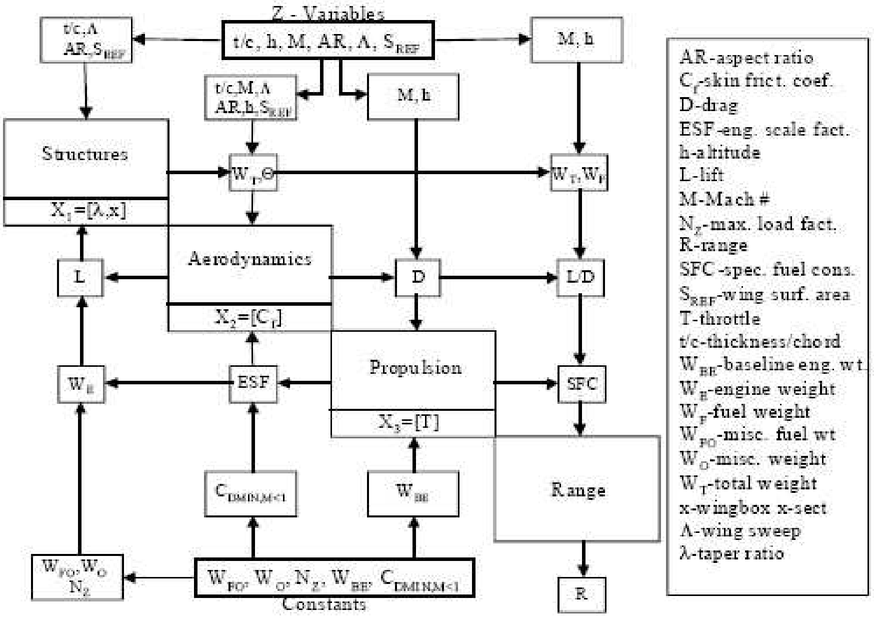

Figure 2: Variable dependency for SSBJ test case[4]

5 CONCLUSIONS

A new approach for multi disciplinary optimization is proposed. The approach differs from existing MDO methods by

Variable	Size	Description	Scope	Level
X1	2 x 1	$[\lambda, x]$	Private	SubSys-1
X2	1 x 1	$[C_f]$	Private	SubSys-2
X3	1 x 1	$[T]$	Private	SubSys-3
Y1	3 x 1	$[W_T, W_F, \Theta]$	Shared/Public	-
Y2	3 x 1	$[L, D, L/D]$	Shared/Public	-
Y3	3 x 1	$[SFC, W_E, ESF]$	Shared/Public	-
Y4	1 x 1	$[R]$	Shared/Public	-
Z	6 x 1	$[t/c, h, M, AR, \Lambda, S_{ref}]$	Public	System Level

Table 4: SSBJ test case: Division of parameters

Parameter	Lower Bound	Upper bound	Initial	Optimal	BLISS Optimal [4]
t/c	0.01	0.09	0.05	0.06	0.06
$h(km)$	30	60	45	60	60
M	1.40	1.80	1.60	1.40	1.4
AR	2.50	8.50	5.50	2.50	2.5
Λ	40.0	70.0	55.0	70.0	70.0
$S_{ref}(m^2)$	500	1500	1000	1500	1500
W_T	-	-	1	44748.5	44753.0234
W_e	-	-	1	19350.6	19350.6
Θ	-	-	1	1.02825	1.03
L	-	-	1	44748.5	44753.024
D	-	-	1	5552.84	5552.85
L/D	-	-	1	8.0587	8.0595
SFC	-	-	1	0.92394	0.92393
W_e	-	-	1	9435.9	9435.62
ESF	-	-	1	0.7327	0.7327
λ	0.10	0.40	0.40	0.40	0.4
x	0.75	1.25	0.75	0.75	0.75
C_f	0.75	1.25	0.75	0.75	0.75
T	0.10	1.00	0.16	0.16	0.16

Table 5: SSBJ test case results

- Its treatement of different design variables involved in the MDO process as public and private parameters.
- To solve the system of equations governing interaction variables as state equations. Thereby permiting use of adjoint method for the solution of state equations.
- Embedded model management framework.

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