J. Rauch
Université de Paris 13

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The Wave Equation and the Laws of Geometric Optics

The physical theory called *Geometric Optics* is at the heart of the design of telescopes, microscopes, cameras, ... etc. The three basic laws of this theory are;

1. Light progagates in straight lines.

2. Reflects at boundaries following the usual law that the angles of incidence and reflection are equal.

3. Refracts at interfaces of materials with different indices of refraction according to Snell's Law.

The **prinicipal goal** of this course is to show how these laws are explained by a wave theory of light. A key ingredient is short wavelength asymptotic analysis. The passage from wave optics to the ray theory is an example of the *wave particle duality*.

Outline

Lecture 1. Present some basic facts about the wave equation, in particular in dimesnsions $d > 1$. This is the model for wave propagation in higher dimensions.

Lectures 2-3. Use the Fourier trasnform to derive short wavelength asymptotic solutions. Inroduce the boundary value problems and transmission problems relevant to the laws of reflection and refraction. Use the short wavelength solutions to explain the three laws of geometric optics.

References

The material in the first lecture is in my book *Partial Differential Equations*, and in others as well. A copy of my book can be consulted in the library of the Département de Mathématiques. Ask the librarian. for the copy which does *not* circulate.

The second and third lecture closely follow §1.3 and §1.5 from the first chapter of a book in preparation. The latter section was written for this course. That chapter is available on my web page (`www.math.lsa.umich.edu/~rauch`). Follow the link to Course Materials. The second group of materials are for this course and contain this outline and the book chapter.

Outline of first lecture

1. D'Alembert's formula for the case $d = 1$.

2. Fourier transform, Fourier Inversion Formula, Parseval Identity.

3. Plane wave solutions.

4. Solution of the Cauchy problem by Fourier transformation.

5. Conservation of energy in Fourier and local identity.

6. Finite speed of propagation.

7. Spherical solutions.